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Algebraic characterizations of generating and affinely generating Γ-magic maps and Γ-distance magic labelings on regular graphs

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For a finite abelian group Γ, a Γ-magic map on a graph G is a map $f\colon V(G) \to \Gamma$ with constant open-neighborhood sums. It is *generating* if its labels generate Γ, and *affinely generating* if their pairwise differences generate Γ. Let G be regular and write

$$\Gamma \cong \bigoplus_{i=1}^{r} \mathbb{Z}/d_i\mathbb{Z}, \qquad d_1 \mid \cdots \mid d_r, \qquad B_\Gamma = \bigoplus_{i=1}^{r-1} \mathbb{Z}/d_i\mathbb{Z},$$

with $B_\Gamma = \{0\}$ when $r = 1$. Let $\overline{A}_m$ be the endomorphism induced by adjacency on $(\mathbb{Z}/m\mathbb{Z})^{V(G)}$ modulo constant vectors. We prove that G admits generating and affinely generating Γ-magic maps if and only if, respectively,

$$B_\Gamma \hookrightarrow \ker \overline{A}_{d_r} \qquad \text{and} \qquad \Gamma \hookrightarrow \ker \overline{A}_{d_r}.$$

When $|V(G)| = |\Gamma|$, a bijective Γ-magic map, called a Γ-distance magic labeling, exists precisely when the latter embedding has an image that separates every pair of distinct vertices by a coordinate difference. If the integral reduced adjacency operator is nonsingular over $\mathbb{Q}$, the first two criteria become subgroup conditions in its cokernel.

A graph of order n is *distance magic* if it admits a bijection from its vertex set to $\{1, \ldots, n\}$ with constant open-neighborhood sums. Cichacz and Froncek conjectured that every such graph is Γ-distance magic for every abelian group Γ of order n. We use these characterizations to disprove this conjecture by constructing a 6-regular distance magic graph of order 27 with no affinely generating $(\mathbb{Z}/3\mathbb{Z})^3$-magic map. For Cayley graphs on elementary abelian p-groups, the same characterizations yield a nonexistence result which, combined with a known sufficient condition, shows that a Γ-distance magic labeling exists over some abelian group Γ of the graph order if and only if the regularity is divisible by p.